Exam 3 Review 11/8/2023

Come by 6:15 PM

Exam 3

Posted Oct 30, 2023 10:38 AM

Exam 3 is Wednesday, November 8 from 6:30pm-7:30pm. The location of the exam will depend on your instructor (and please note that these locations are different than the first 2 exams):

Ben Doyle: LILY 1105

Victor Hughes: CL50 224

Jakayla Robbins: LILY 1105

Alexandra Cuadra: CL50 224

Dave Norris: CL50 224

There is an exam memo in Brightspace under Contents->Exam Information with more detailed information about the exam. You will be emailed a seating assignment closer to the exam date.

Quiz Friday 11/10/23

f_{xx} f_{xy} f_{yx} f_{yy}

A radioactive substance has a half-life of $t_{1/2} = 10$ years. The rate at which the substance decays is given by $\frac{dN}{dt} = kN$ where N is the number of grams present at time t in years. There are 25 grams present at $t = 0$. How many grams of the substance remains after $t = 5$ years?

- ○ $\frac{25}{\sqrt{5}}$ grams
- ○ $\frac{25}{2}$ grams
- ○ $\frac{25}{\sqrt{10}}$ grams
- ○ $\frac{25}{\sqrt{2}}$ grams
- ○ $\frac{75}{4}$ grams

0 $N(0) = 25$

$N(10) = 12.5$

1

$$\frac{dN}{dt} = kN$$

$$\int \frac{dN}{N} = \int k\,dt$$

$$\ln(N) = kt + C$$

$$N = e^{kt+C}$$

$$N = e^{kt}\,e^{C}$$

constant A

$$N = Ae^{kt}$$

2 $N(0) = 25$

$$25 = Ae^{k\cdot 0}$$

$$25 = A\cdot 1$$

$$A = 25$$

$$N = 25e^{kt}$$

$$N(10) = 12.5$$

$$\frac{12.5}{25} = \frac{25e^{k(10)}}{25}$$

$$\frac{1}{2} = e^{k(10)}$$

$$\ln\left(\tfrac{1}{2}\right) = k\cdot 10$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{10}$$

$$a^{krs} \quad \ne \left(\left(a^{k}\right)^{r}\right)^{s}$$

3 $N = 25e^{\frac{\ln(\frac{1}{2})}{10}t}$

$$N = 25\,e^{\left[\ln(\frac{1}{2})/10\right]\cdot 5}$$

$$= 25\left(\left(e^{\ln(\frac{1}{2})}\right)^{1/10}\right)^{5}$$

$$= 25\left(\frac{1}{2}\right)^{5/10} = 25\left(\frac{1}{2}\right)^{1/2}$$

$$= 25\sqrt{\frac{1}{2}} = \frac{25\cdot\sqrt{1}}{\sqrt{2}}$$

$$= \frac{25}{\sqrt{2}}$$

What are the first 3 non-zero terms of the Maclaurin series representation of the following?

$$\int \cos \sqrt{x}\, dx$$

- $x - \frac{x^2}{4} + \frac{x^3}{72}$
- $x - \frac{x^3}{6} + \frac{x^5}{120}$
- $\sqrt{x} - \frac{x}{2} + \frac{x^{3/2}}{24}$
- $1 - \frac{x^2}{2} + \frac{x^4}{24}$
- $1 - \frac{x}{2} + \frac{x^2}{24}$
- $1 - \frac{\sqrt{x}}{2} + \frac{x}{24}$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\cos(\sqrt{x}) \approx 1 - \frac{(\sqrt{x})^2}{2!} + \frac{(\sqrt{x})^4}{4!}$$

$$= 1 - \frac{x}{2} + \frac{x^2}{24}$$

$$\int \cos\sqrt{x}\, dx \approx \int \left(1 - \frac{1}{2}x + \frac{1}{24}x^2\right) dx$$

$$= x - \frac{1}{2}\frac{x^2}{2} + \frac{1}{24}\frac{x^3}{3}$$

$$= x - \frac{x^2}{4} + \frac{1}{72}x^3$$

Suppose that $y' = ky$, $y(0) = 9$, and $y'(0) = 7$. What is y as a function of t ?

$y =$ [ ]

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$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln(y) = kt + C$$

$$y = e^{kt+C}$$

$$y = Ae^{kt}$$

$y(0) = 9$

$y'(0) = 7$

$y(0) = 9$

$9 = Ae^{k \cdot 0}$

$9 = A$

$y(t) = 9e^{kt}$

2 options for using $y'(0) = 7$

Option 1:

$y'(t) = 9e^{kt} \cdot k$

$7 = 9\underbrace{e^{k \cdot 0}}_{1} \cdot k$

$k = \frac{7}{9}$

Option 2:

$y' = ky$

$y(0) = 9$

$y'(0) = 7$

@ $t = 0$ $\quad 7 = k \cdot 9$

$k = \frac{7}{9}$

Ans: $y = 9e^{\frac{7}{9}t}$

Find the general solution of the given differential equation. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 7e^{-4x-y}$$

- $y = -4x\ln\left(\frac{7}{4}\right) + C$
- $y = -\frac{7}{4}e^{-4x} + C$
- $y = -4x\ln(7) + C$
- $y = 7x + C$
- $y = \ln\left(\frac{7}{4}e^{-4x} + C\right)$
- $y = \ln\left(-\frac{7}{4}e^{-4x} + C\right)$

$u = mx + b$

$\frac{1}{m}du = dx$

$$\frac{dy}{dx} = 7e^{-4x}e^{-y}$$

$$\int \frac{dy}{e^{-y}} = \int 7e^{-4x}\,dx$$

$$\int e^{y}dy = \int 7e^{-4x}dx$$

$u = -4x$

$$e^{y} = 7 \cdot \frac{1}{-4}e^{-4x} + C$$

In a particular chemical reaction, a substance is converted into a second substance at a rate proportional to the square of the amount of the first substance present at any time, t. Initially, 58 grams of the first substance was present, and 1 hour later only 15 grams of the first substance remained. What is the amount of the first substance remaining after 5 hours? (Round your answer to four decimal places.)

$m =$ ______ grams

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$$\frac{dA}{dt} = kA^2$$

$A(0) = 58$

$A(1) = 15$

$A(5) = ?$

$$\int \frac{dA}{A^2} = \int k\,dt$$

$$\int A^{-2}\,dA = \int k\,dt$$

$$\frac{A^{-1}}{-1} = kt + C$$

$$\frac{-1}{A} = kt + C$$

$$\frac{-1}{58} = k(0) + C$$

$$C = \frac{-1}{58}$$

$$\frac{-1}{A} = kt - \frac{1}{58}$$

$$\frac{-1}{15} = k(1) - \frac{1}{58}$$

$$k = \frac{-1}{15} + \frac{1}{58} = \frac{-43}{(15)(58)}$$

$$\frac{-1}{A} = \frac{-43}{(15)(58)} \cdot 5 - \frac{1}{58}$$

Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

Find the sum....

Hint - start writing series so you can find 1st term (a) and common ratio (r).
Then sum is $\frac{a}{1-r} = \frac{\text{1st term}}{1 - \text{common ratio}}$

- $\frac{2}{33}$
- $\frac{2}{21}$
- $\frac{3}{11}$
- $\frac{6}{11}$
- $\frac{3}{7}$
- $\frac{8}{3}$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \frac{1}{3} + \frac{-2}{3^3} + \frac{(-2)^2}{3^5} + \frac{(-2)^3}{3^7}$$

1st term $= \frac{1}{3} = a$

com. ratio $= \frac{-2}{3^2} = \frac{-2}{9}$

$$\text{sum} = \frac{1/3}{1-(-2/9)} = \frac{1/3}{11/9} = \frac{1}{\cancel{3}} \cdot \frac{\overset{3}{\cancel{9}}}{11} = \frac{3}{11}$$